Confidence Intervals for the Mean of Lognormal Distribution with Restricted Parameter Space

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Abstract

This paper presents new confidence intervals for the mean of lognormal distribution with restricted parameter. We proved the coverage probability and expected length of our proposed confidence interval. Monte Carlo simulation will be used to compare the proposed confidence interval to the existing confidence interval.

Mathematics Subject Classification: 62F25

Keywords: Lognormal distribution, the mean, restricted parameter space

1 Introduction

Let $X = (X_1, X_2, ..., X_n)$ be a random variable having a lognormal distribution, and μ and σ^2 , respectively, are denoted by the mean and the variance of Y where $Y = \ln(X) \sim N(\mu, \sigma^2)$. The probability density function of the lognormal distribution, $LN(\mu, \sigma^2)$, is

$$f(x,\mu,\sigma^2) = \begin{cases} \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right) & ; \text{ for } x > 0\\ 0 & ; \text{ for } x \le 0. \end{cases}$$
 (1)

The mean for the lognormal population is $E(X) = \exp(\mu + \sigma^2/2)$ where E(X) denotes the expectation of X. We are interesting to construct the confidence interval of $\theta^* = \exp(\mu + \sigma^2/2)$ when parameter θ^* is bounded i.e., $a < \theta^* < b$ where a and b are constant and a < b.