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# A Single-Level Continuous Sampling Plan for High Quality Production Line

**Tidadeaw Mayureesawan**
Department of Applied Statistics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok, Thailand.
E-mail: tms@kmutnb.ac.th

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**Abstract**

This paper presents a plan SKIP-CSP-1 for inspection of a high quality continuous production line. The plan is defined by 3 parameters $i$ (the number of consecutive non-defective units that must be produced during a 100% inspection of the line), a fraction $f$ (the specified sampling frequency during a fractional inspection of the line) and $k$ (the number of units for skipping over in an inspection). SKIP-CSP-1 computes 3 performance measures, average fraction inspected ( $AFI$ ), average outgoing quality ( $AOQ$ ) and average outgoing quality limit ( $AOQL$ ), for given values of the parameters and incoming fraction of defective units on the line ( $p$ ). The validity of the performance measure formulas have been tested by extensive simulations. The formulas of performance measures, $AFI$ and $AOQ$ are valid for all the sets of $p, i, k, r$ ( $r=1/f$ ) values. The SKIP-CSP-1 plan has been compared with CSP-1 and CSP-2 plans. On comparing $AFI$ and $AOQ$, we have found that SKIP-CSP-1 does not give an appreciable difference in the number of units inspected and output quality for low level of $p$ (0.001, 0.003, 0.005) and for all the sets of $i, k, r$. Further, for higher levels of $p$, but low levels of $k$, it also does not give different results. However, compared with CSP-1, we have found that SKIP-CSP-1 gives lower number of units inspected and output quality than CSP-1, whereas compared with CSP-2, SKIP-CSP-1 gives higher number of units inspected and output quality than CSP-2.

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**Keywords:** continuous sampling plan, high quality production line.

## 1. Introduction

A continuous sampling plan (CSP) is a plan of sampling inspection for a product consisting of individual units (parts, subassemblies, finished articles etc.) that is manufactured in quantity by an essentially continuous process [1]. A CSP is applicable only to units subject to nondestructive inspection on a GO-NOGO basis. It is intended primarily for use in process inspection of parts, or final inspection of finished articles, where it is desired to have assurance that the percentage of defective units in the accepted product will be less than some prescribed low figure. The original continuous sampling plan (CSP-1) was described by H.F. Dodge and variations of the plan (e.g., CSP-1, CSP-2, CSP-2 CSP-4, CSP-5, CSP-F), have been proposed by many workers.

An inspection procedure in CSP-1 always starts with 100% inspection (screening). This screening is performed until $i$ successive non-defective units are observed. Then the procedure samples only one of the following $r$ units. If the sampled unit is found to be good, then the procedure continues to sample one unit from the next $r$, etc. As soon as a defective sample unit is observed, the procedure switches back to screening every unit and continues until a further $i$ consecutive good units have been observed. Thus, the inspection procedure consists of alternating periods of 100% inspection and periods with $f.100\%$ inspection, where $f = 1/r$ [2].

A CSP-2 differs from a CSP-1 in the sense that during the sampling inspection period the first observed defective unit does not immediately require the procedure to change to 100% nspection. Switching only takes place if another defective is found in the following m sampled units. Frequently, one chooses $m = i$. This choice of $m$ implies that, after discovery of the first nonconforming unit during the $f.100\%$ inspection period, the inspection needs to draw only good units in the next $i$ sampled units [3].

Reviews of other CSPs are now available in textbooks (see, e.g., Duncan [4], Grant [5] and Montgomery [6]).

The main objective of this paper is to develop a CSP that can be used for a high quality production line. The paper describes the following:

1) The design of a continuous sampling plan for a high quality production line which we call SKIP-CSP-1
2) The development of the theory and formulas for important performance measures in SKIP-CSP-1, such as the average fraction inspected ( $AFI$ ), the average outgoing quality ( $AOQ$ ) and the average outgoing quality limit ( $AOQL$ ).
3) Tests of the validity of the formulas for the performance measures by comparison of the values computed from the formulas with values obtained through extensive simulations.
4) A comparison of values of the performance measures of the SKIP-CSP-1 plan with the CSP-1 and CSP-2 plans.

## 2. Materials and Methods

### 2.1 The operating procedure of SKIP-CSP-1

Assume that inspection is to be made for only one quality characteristic, so that interest will be centered on one kind of defect. The SKIP-CSP-1 uses 3 parameters for inspection of the units being produced on the production line, namely 2 positive integers $i$ and $k$, and a fraction $f$, which are defined by:

$i$ A number of consecutive non-defective units that must be produced during a 100% inspection of units produced on the line.

$f$ A sampling frequency for a fractional inspection of units produced on the line ( $f=1/r$ ).

$k$ A number of units for skipping over in the inspection.

The units sampled during a fractional $f$ inspection of a line must be an unbiased sample of units produced on the line. In all inspection schemes any defective unit that is detected will be replaced immediately by a non-defective unit.

The procedure for inspection of the SKIP-CSP-1 is as follows:

(1) Inspect 100% of the units consecutively as produced and continue such inspection until $i$ units in succession are found clear of defects. When $i$ units in succession are found clear of defects, discontinue 100% inspection.

(2) After discontinuing 100% inspection, the selection of the inspection scheme for the new step depends on the results of the preceding step. The selection rules are as follows:

(2.1) If the number of units inspected consecutively in a 100% inspection of the units is equal to $i$ (no defective units were detected when inspecting 100% of the first $i$ units consecutively), the next $k$ units in succession are skipped over (not inspected) before going on to step 3.

(2.2) If the number of units inspected consecutively in a 100% inspection of the units is greater than $i$ (at least one defective unit is detected on the line before $i$ units consecutively are found clear of defects), go on to step 3.

(3) Inspect only a fraction $f$ of the units, selecting individual sample units one at a time from the product flow. This scheme continues until a defective unit is found on the line.

(4) If a sample unit is found defective, revert immediately to a 100% inspection of succeeding units and return to step (1).

In summary, a 100% inspection on the line must continue until the specified number $i$ of consecutive non-defective units are produced on the line. A successful 100% inspection on a line will be followed by an $f$ inspection on the line. In an $f$ inspection, if a defective unit is detected then the plan reverts to a 100% inspection on the line, as in the procedure for inspection of the CSP-1 plan. The different procedure for inspection of SKIP-CSP-1 from CSP-1 is that if a 100% inspection does not find any defects in the first $i$ units inspected then no inspections will be carried out for the next $k$ units.

**2.2 The performance measures used in SKIP-CSP-1**

The performance measures that we define in SKIP-CSP-1 are generalizations of the performance measures $AFI$ (average fraction inspected), $AOQ$ (average outgoing quality) and $AOQL$ (average outgoing quality limit) used in the conventional CSPs [2]. The measures that we use are as follows:

- The average fraction inspected ( $AFI$ ).
- The average outgoing quality ( $AOQ$ ).
- The average outgoing quality limit ( $AOQL$ ).

### 2.2.1 The average fraction inspected

We have identified 2 phases for the inspection process of the SKIP-CSP-1. Phase 1 consists of a 100% inspection followed by a possible $k$ units with zero inspection. Phase 2 is an $f$ inspection on the line. The average cycle length is the total of the average number of units produced on the line during the inspections of phase 1 and phase 2. The average total inspection is the total of the average number of units inspected on the line during the inspections of phase 1 and phase 2.

We are concerned with the average spacing between defective units on the line. The probability of producing a defective unit is defined to be $p$. The events of particular interest are a sequence of $d$ non-defective units ($0 \leq d < i$) followed by a defective unit. The complete set of such probabilities for all possible sequences, having respectively $i = 0, 1, 2, \ldots \infty$, defines a probability distribution of random order spacing of defects in uniform product. This is shown in Table 1 in which O represents a non-defective unit, X represents a defective one and $q = 1 - p$. These probabilities are the successive terms in the infinite power series

$$p + pq + pq^2 + pq^3 + \ldots = p(1 + q + q^2 + q^3 + \ldots) \tag{1}$$

**Table 1.** The complete set of probabilities for possible sequences that define a probability distribution of random order spacing of defects on the production line.

| Sequence | X | OX | OOX | OOOX | ... | OOO...OX | ... |
|---|---|---|---|---|---|---|---|
| No. of Term in the Power Series | 1 | 2 | 3 | 4 | ... | $i$ | ... |
| No. of Non-defective Units before Finding the Next Defect | 0 | 1 | 2 | 3 | ... | $i$-1 | ... |
| Probability of Occurrence | $p$ | $pq$ | $pq^2$ | $pq^3$ | ... | $pq^{i-1}$ | ... |

The sum of the first $i$ terms is the probability, $A$, of failing to find the next $i$ units clear of defects, which is

$$A = \sum_{n=0}^{i-1} pq^n = \sum_{n=0}^{i-1} (1-q)\, q^n = 1 - q^i . \tag{2}$$

In turn, the sum of all terms beyond the $i$ th term is the probability of finding 0 defects in the next $i$ units, which is

$$B = 1-A = q^i . \tag{3}$$

The average fraction inspected in the long run is defined by:

$$AFI = \frac{ATI}{ACL} \tag{4}$$

where

$ATI$ = The average number of units inspected on the line during the inspections of phase 1 and phase 2 (Average total inspection),

$ACL$ = The average number of units produced on the line during the inspections of phase 1 and phase 2 (Average cycle length).

**The average number of units produced on the line during phase 1**

We define $l$ as the average number of units produced on the line during phase 1. When inspect 100% of the units consecutively as produced on the line, 2 cases can be happened. For case 1 is the number of units inspected consecutively in a 100% inspection of the units is equal to $i$ (no defective units were detected when inspecting 100% of the first $i$ units consecutively). The next $k$ units in succession are skipped over (not inspected) before inspect only a fraction $f$ of the units. Then $i+k$ of units that will be passed with the probability, $B$ as define in (3). For case 2 is the number of units inspected consecutively in a 100% inspection of the units is greater than $i$ (at least one defective unit is detected on the line before $i$ units consecutively are found clear of defects). We define $h$ as the average number of units inspected in this failure sequence. The average $h$ is

$$h = \frac{p}{1-q^i}(1+2q+3q^2+4q^3+...+iq^{i-1})$$

This may be eva uated as follows:

$$h = \frac{p}{1-q^i}\left(\frac{1-q^i(1+pi)}{p^2}\right)$$

$$= \frac{1}{p(1-q^i)}\;[1-q^i(1+pi)] . \tag{5}$$

The next step is to determine the average number of failure sequences that will be encountered before finding $i$ units clear of defects. This average number designated as $G$, may be found from the probability distribution of all possible numbers of failure sequences, expressed by the infinite series

$$B\,(1+A+A^2+A^3+\ldots)\,. \tag{6}$$

The successive terms are the probabilities of occurrence of 0, 1, 2, 3, etc. failure sequences before finding $i$ units clear of defects. $G$ is given by the sum of the infinite series

$$G \;=\; B\,(0+1A+2A^2+3A^3+\ldots) \;=\; B\,A(1+2A+3A^2+4A^3+\ldots)$$

Summing the series, we have
$$G \;=\; BA\frac{1}{(1-A)^2} \;=\; \frac{1-q^i}{q^i}\,. \tag{7}$$

The average number of units produced on the line during the inspections is $Gh+i$ units with probability of this case, $A$ as define in (2), and it is therefore

$$Gh+i \;=\; \left(\frac{1-q^i}{q^i}\right)\left(\frac{1}{p(1-q^i)}\,[1-q^i\,(1+pi)]\right)+i$$
$$=\; \frac{1-q^i}{pq^i}\,. \tag{8}$$

Now $l$ is the average number of units produced on the line during phase 1. We have

$$l \;=\; (i+k)B+(Gh+i)A$$
$$=\; (i+k)(1-p)^i+\frac{(1-(1-p)^i)^2}{p(1-p)^i}\,. \tag{9}$$

**The average number of units produced on the line during phase 2**

We define $v$ as the average number of units produced on the line during phase 2 before a defect is found. $v$ will be $1/f$ times the average number of individual sample units inspected in such a period. The average number of sample units inspected in a period of sampling inspection will thus be the average defect spacing for product having fractional defective, $p$, and therefore it is given by the infinite series:

$$H = p\,(1+2q+3q^2+4q^3+...)$$

Summing the series, we have $$H = \frac{p}{(1-q)^2} = \frac{1}{p}, \qquad (10)$$

$$v = \frac{1}{f}H = \frac{1}{f\,p}. \qquad (11)$$

**The average cycle length**

The average cycle length (*ACL*) is the sum of the average number of units produced on the line during the inspections of phase 1 and phase 2, we have

$$ACL = l+v \qquad (12)$$

where $l$ and $v$ as define in (9) and (11) respectively.

**The average number of units inspected on the line during phase 1**

We define $u$ as the average number of pieces inspected during a 100% inspection followed by a possible $k$ units with zero inspection. We have

$$u = iB+(Gh+i)A$$

$$= i\,(1-p)^i + \frac{(1-(1-p)^i)^2}{p\,(1-p)^i}. \qquad (13)$$

**The average number of units inspected on the line during phase 2**

For phase 2, the average number of units inspected is $f\,v$ for fractional sampling inspections.

**The average total inspection**

The average total inspection ( $ATI$ ) is the sum of the average number of units inspected on the line during the inspections of phase 1 and phase 2, we have

$$ATI = u+f\,v. \qquad (14)$$

**2.2.2 The average outgoing quality**

Suppose a SKIP-CSP-1 is selected by choosing specific values of $i, k$ and $r$. The average fraction of defective units that appear in the final output of line is called the average outgoing quality ( $AOQ$ ). The $AOQ$ is the average fraction of defective units that occur on the line multiplied by the average fraction of units not inspected (recall that an inspected unit that is found to be defective is immediately replaced by a non-defective unit). The average fraction of input defective units is $p$ and, the average fraction not inspected is $(1-AFI)$. Therefore, we have that $AOQ$ is given by:

$$AOQ = p(1-AFI). \tag{15}$$

### 2.2.3 The average outgoing quality limit

For given values of $i, k$ and $r$, the $AOQ$ will have a maximum for some particular incoming fractions of defective units on the line. This value is $AOQL$ defined by:

$$AOQL = \underset{p}{Max}(AOQ). \tag{16}$$

## 2.3 Tests of the Validity of Performance Measures for SKIP-CSP-1

In order to test the validity of the performance measures that we have defined for SKIP-CSP-1, we have compared the results from our formulas with the values for the performance measures obtained from extensive simulations. We have examined 7 different levels for the incoming fractions $p$ of defective units produced on the line 0.001, 0.003, 0.005, 0.008, 0.01, 0.03, 0.05. For each $p$ we selected values of $i$ = 5, 10, 15, 20, 25, values of $r$ = 2, 3, and values of $k \leq i$ (3, 5, 10, 15, 20, 25).

For each set of values of the constants $p, i, k, r$, we carried out a simulation to compute the fraction of units inspected, and the fraction of outgoing defective units. The simulation was repeated 500 times and the values of $AFI$ and $AOQ$ were calculated. These values were then compared with the values of $AFI$ and $AOQ$ computed from the formulas given in equations (1) and equation (5) respectively.

We accepted a formula as a valid formula if the percentage difference between the $AFI$ value from the formula and the $AFI$ value from the simulations was less than or equal to 2. In testing the validity of an $AOQ$ formula, we accepted the formula as a valid formula if the percentage difference between the $AOQ$ value from the formula and the $AOQ$ value from the simulations was less than or equal to 2. We then compared the validity of the formulas for each set of $p, i, k, r$ values. The results for the comparisons are summarized in section 3.1.

**2.4 Comparisons of the performance measure of the SKIP-CSP-1 plan with CSP-1 and CSP-2 plan.**

In this section we compare the average fraction inspected ($AFI$) values and the average outgoing quality ($AOQ$) values for SKIP-CSP-1 with $AFI$ and $AOQ$ values respectively obtained for CSP-1 and CSP-2. We have carried out extensive simulations for the three inspection schemes using the same parameter values that we used for testing the validity of the performance measures in SKIP-CSP-1.

We have defined the $\%DIFF\_AFI$ values for comparing the $AFI$ values of the SKIP-CSP-1 plan with the CSP-1 plan by:

$$\%DIFF\_AFI = \frac{[(AFI\_CSP\text{-}1) - (AFI\_SKIP\text{-}CSP\text{-}1)] \times 100}{AFI\_SKIP\text{-}CSP\text{-}1}, \tag{17}$$

where

$AFI\_SKIP\text{-}CSP\text{-}1$ = the $AFI$ values of SKIP-CSP-1 plan,

$AFI\_CSP\text{-}1$ = the $AFI$ values of CSP-1 plan,

$AFI\_CSP\text{-}2$ = the $AFI$ values of CSP-2 plan,

and have compared the $AFI$ values of the SKIP-CSP-1 plan with the CSP-2 plan by replacing $AFI\_CSP\text{-}1$ as in (17) with $AFI\_CSP\text{-}2$.

We have defined the $\%DIFF\_AOQ$ values for comparing the $AOQ$ values of the SKIP-CSP-1 plan with the CSP-1 plan by:

$$\%DIFF\_AOQ = \frac{[(AOQ\_SKIP\text{-}CSP\text{-}1) - (AOQ\_CSP\text{-}1)] \times 100}{AOQ\_SKIP\text{-}CSP\text{-}1}, \tag{18}$$

where

$AOQ\_SKIP\text{-}CSP\text{-}1$ = the $AOQ$ values of SKIP-CSP-1 plan,

$AOQ\_CSP\text{-}1$ = the $AOQ$ values of CSP-1 plan,

$AOQ\_CSP\text{-}2$ = the $AOQ$ values of CSP-2 plan.

and for comparing the $AOQ$ values of the SKIP-CSP-1 plan with the CSP-2 plan by replacing $AOQ\_CSP\text{-}1$ as in (18) with $AOQ\_CSP\text{-}2$.

The results for the comparisons are summarized in section 3.2.

### 3. Results

#### 3.1 The Validity of Performance Measures for SKIP-CSP-1

In Table 2 we show, for each choice of $p, i, k, r$ values, the percentage differences of the $AFI$, $AOQ$ values from the formula and the $AFI$, $AOQ$ values from the simulations. We found that the percentage differences were less than 2 for all sets of $p$, $i$, $k$ and $r$ values. Our simulations indicated that the $AFI$ and $AOQ$ formulas are valid.

**Table 2.** The percentage differences between the $AFI$, $AOQ$ values from the formula and the $AFI$, $AOQ$ values from the simulations for SKIP-CSP-1.

| *i* | *k* | *r* | *p* = 0.001 | | *p* = 0.003 | | *p* = 0.005 | | *p* = 0.008 | | *p* = 0.01 | | *p* = 0.03 | | *p* = 0.05 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | *AFI* | *AOQ* | *AFI* | *AOQ* | *AFI* | *AOQ* | *AFI* | *AOQ* | *AFI* | *AOQ* | *AFI* | *AOQ* | *AFI* | *AOQ* |
| 5 | 3 | 2 | 0.21 | 1.89 | 0.09 | 0.70 | 0.09 | 0.06 | 0.13 | 0.80 | 0.05 | 1.07 | 0.98 | 1.19 | 0.11 | 0.91 |
| 5 | 3 | 3 | 0.40 | 0.89 | 0.59 | 1.20 | 0.58 | 0.06 | 0.85 | 0.65 | 0.01 | 0.12 | 0.58 | 0.28 | 0.14 | 0.45 |
| 10 | 10 | 2 | 0.00 | 0.14 | 0.03 | 0.87 | 0.33 | 1.04 | 0.19 | 0.68 | 0.46 | 0.40 | 0.57 | 0.45 | 1.10 | 0.04 |
| 10 | 10 | 3 | 0.25 | 0.62 | 0.10 | 0.28 | 0.36 | 0.34 | 1.15 | 0.37 | 0.21 | 0.79 | 0.20 | 1.09 | 1.42 | 0.55 |
| 10 | 5 | 2 | 0.19 | 0.36 | 0.03 | 1.23 | 0.06 | 0.36 | 0.09 | 0.18 | 0.34 | 0.74 | 0.64 | 0.92 | 1.08 | 0.51 |
| 10 | 5 | 3 | 0.22 | 0.92 | 0.38 | 0.38 | 0.48 | 0.69 | 1.27 | 0.33 | 0.39 | 0.57 | 0.21 | 1.01 | 1.22 | 1.28 |
| 15 | 15 | 2 | 0.04 | 0.51 | 0.05 | 0.91 | 0.08 | 0.17 | 0.01 | 0.69 | 1.00 | 1.23 | 1.38 | 0.60 | 1.48 | 0.25 |
| 15 | 10 | 2 | 0.07 | 1.30 | 0.02 | 1.01 | 0.03 | 0.31 | 0.68 | 1.45 | 0.76 | 1.24 | 1.41 | 1.04 | 0.22 | 0.53 |
| 15 | 5 | 2 | 0.16 | 1.13 | 0.28 | 0.39 | 0.18 | 1.35 | 0.24 | 0.68 | 0.81 | 1.32 | 0.62 | 1.40 | 0.41 | 0.76 |
| 15 | 15 | 3 | 0.18 | 1.89 | 0.39 | 1.18 | 0.26 | 0.15 | 1.05 | 0.10 | 0.15 | 0.29 | 1.16 | 0.27 | 0.71 | 1.10 |
| 15 | 10 | 3 | 0.11 | 0.24 | 0.06 | 0.68 | 0.21 | 0.56 | 0.22 | 1.44 | 0.68 | 0.92 | 0.91 | 1.43 | 0.68 | 1.43 |
| 15 | 5 | 3 | 0.01 | 1.72 | 0.73 | 0.02 | 0.59 | 1.04 | 0.92 | 1.25 | 0.38 | 0.83 | 0.70 | 0.88 | 0.03 | 0.53 |
| 20 | 20 | 2 | 0.01 | 0.46 | 0.17 | 0.54 | 0.46 | 0.11 | 1.13 | 1.09 | 1.47 | 0.35 | 0.76 | 0.41 | 0.16 | 0.36 |
| 20 | 15 | 2 | 0.15 | 1.73 | 0.05 | 1.12 | 0.03 | 1.52 | 0.62 | 0.59 | 0.84 | 0.31 | 1.36 | 0.96 | 1.33 | 0.24 |
| 20 | 10 | 2 | 0.01 | 0.20 | 0.18 | 0.21 | 1.26 | 1.17 | 0.86 | 1.26 | 0.84 | 0.31 | 1.20 | 0.03 | 0.27 | 0.67 |
| 20 | 5 | 2 | 0.01 | 1.60 | 0.00 | 0.94 | 0.01 | 1.20 | 0.00 | 0.94 | 1.39 | 1.09 | 0.83 | 0.54 | 0.41 | 0.89 |
| 20 | 20 | 3 | 0.16 | 1.86 | 0.40 | 0.30 | 0.13 | 0.47 | 1.04 | 1.26 | 0.55 | 1.44 | 1.24 | 1.10 | 0.35 | 1.04 |
| 20 | 15 | 3 | 0.01 | 1.94 | 0.00 | 0.29 | 0.85 | 1.33 | 1.03 | 0.13 | 0.65 | 0.42 | 1.04 | 1.33 | 0.23 | 0.10 |
| 20 | 10 | 3 | 0.05 | 1.20 | 0.85 | 0.06 | 0.90 | 1.55 | 0.29 | 0.35 | 0.86 | 0.83 | 1.45 | 1.05 | 0.35 | 1.32 |
| 20 | 5 | 3 | 0.11 | 1.28 | 0.01 | 0.01 | 0.08 | 0.68 | 0.44 | 1.44 | 0.68 | 0.30 | 0.44 | 1.04 | 1.47 | 0.83 |
| 25 | 25 | 2 | 0.03 | 0.77 | 0.26 | 0.71 | 0.42 | 1.11 | 1.05 | 0.18 | 0.54 | 0.33 | 1.24 | 1.44 | 0.39 | 0.92 |
| 25 | 20 | 2 | 0.11 | 0.85 | 0.05 | 1.18 | 0.16 | 1.78 | 0.63 | 0.92 | 0.95 | 0.40 | 0.95 | 0.18 | 0.24 | 0.88 |
| 25 | 15 | 2 | 0.14 | 0.66 | 0.00 | 1.12 | 0.24 | 1.34 | 0.13 | 1.19 | 1.10 | 1.52 | 1.46 | 0.79 | 0.03 | 0.03 |
| 25 | 10 | 2 | 0.02 | 1.08 | 0.04 | 0.77 | 0.05 | 1.47 | 0.07 | 1.21 | 0.43 | 1.43 | 0.58 | 0.37 | 0.55 | 0.59 |
| 25 | 5 | 2 | 0.10 | 1.45 | 0.04 | 0.90 | 0.18 | 1.49 | 0.66 | 1.37 | 0.99 | 1.67 | 0.89 | 0.41 | 0.19 | 0.11 |
| 25 | 25 | 3 | 0.01 | 0.42 | 0.19 | 0.03 | 0.98 | 1.23 | 0.52 | 0.91 | 0.62 | 1.05 | 0.24 | 1.05 | 1.02 | 0.38 |
| 25 | 20 | 3 | 0.13 | 0.52 | 0.58 | 1.56 | 0.94 | 0.04 | 0.07 | 0.72 | 0.15 | 0.34 | 0.37 | 1.29 | 0.00 | 0.90 |
| 25 | 15 | 3 | 0.10 | 1.55 | 0.06 | 0.25 | 0.28 | 1.46 | 0.03 | 0.65 | 0.15 | 0.35 | 1.27 | 0.74 | 0.27 | 0.06 |
| 25 | 10 | 3 | 0.26 | 0.89 | 0.01 | 0.53 | 1.06 | 1.08 | 0.15 | 0.93 | 0.96 | 1.02 | 0.60 | 0.89 | 0.44 | 0.65 |
| 25 | 5 | 3 | 0.36 | 0.82 | 0.41 | 1.24 | 1.10 | 0.95 | 0.50 | 0.21 | 0.85 | 0.70 | 1.09 | 1.36 | 0.57 | 0.71 |

### 3.2 The comparison of performance measures

In this section, we give the results of the comparison of the average fraction of units inspected using our SKIP-CSP-1 plan with the average fraction inspected using the CSP-1 and CSP-2 plans, and the results of the comparison of the average fraction of defective units outgoing from inspection using our SKIP-CSP-1 plan with the average fraction of defective units outgoing from inspection using the CSP-1 and CSP-2 plans. The results for the $AFI$ values and $AOQ$ values for SKIP-CSP-1, CSP-1 and CSP-2 for $p$ = 0.001 are shown in Table 3, for $p$ = 0.003 are shown in Table 4, and for $p$ = 0.05 are shown in Table 5.

**Table 3.** The $AFI$ values and $AOQ$ values for the SKIP-CSP-1, CSP-1 and CSP-2 plans for $p$ = 0.001

| *i* | *k* | *r* | *AFI* | | | $\%DIFF\_AFI$ | | *AOQ* | | | $\%DIFF\_AOQ$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | SKIP | 1 | 2 | SKIP-1 | SKIP-2 | SKIP | 1 | 2 | SKIP-1 | SKIP-2 |
| 5 | 3 | 2 | 5.0E-01 | 5.0E-01 | 5.0E-01 | 1.5E-01 | -9.9E-02 | 4.9E-04 | 5.0E-04 | 5.0E-04 | 1.5E-01 | -9.9E-02 |
| 5 | 3 | 3 | 3.4E-01 | 3.3E-01 | 3.3E-01 | 1.0E-01 | -2.3E-01 | 6.7E-04 | 6.7E-04 | 6.7E-04 | 5.0E-02 | -1.2E-01 |
| 10 | 10 | 2 | 5.0E-01 | 5.0E-01 | 5.0E-01 | 5.0E-01 | -2.7E-07 | 5.0E-04 | 5.0E-04 | 5.0E-04 | 5.0E-01 | -2.7E-07 |
| 10 | 5 | 2 | 5.0E-01 | 5.0E-01 | 5.0E-01 | 2.5E-01 | -2.5E-01 | 5.0E-04 | 5.0E-04 | 5.0E-04 | 2.5E-01 | -2.5E-01 |
| 10 | 10 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 3.3E-01 | -3.3E-01 | 6.6E-04 | 6.6E-04 | 6.7E-04 | 1.7E-01 | -1.6E-01 |
| 10 | 5 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 1.7E-01 | -4.9E-01 | 6.7E-04 | 6.6E-04 | 6.7E-04 | 8.5E-02 | -2.5E-01 |
| 15 | 15 | 2 | 5.0E-01 | 5.0E-01 | 5.0E-01 | 7.4E-01 | -1.3E-06 | 5.0E-04 | 5.0E-04 | 5.0E-04 | 7.4E-01 | -1.3E-06 |
| 15 | 10 | 2 | 5.0E-01 | 5.0E-01 | 5.0E-01 | 4.9E-01 | -2.4E-01 | 4.9E-04 | 5.0E-04 | 5.0E-04 | 5.0E-01 | -2.4E-01 |
| 15 | 5 | 2 | 5.0E-01 | 5.0E-01 | 5.0E-01 | 2.5E-01 | -4.9E-01 | 4.9E-04 | 5.0E-04 | 5.0E-04 | 2.5E-01 | -4.9E-01 |
| 15 | 15 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 5.0E-01 | -4.9E-01 | 6.8E-04 | 6.6E-04 | 6.7E-04 | 2.5E-01 | -2.4E-01 |
| 15 | 10 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 3.3E-01 | -6.5E-01 | 6.6E-04 | 6.6E-04 | 6.7E-04 | 1.7E-01 | -3.3E-01 |
| 15 | 5 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 1.7E-01 | -8.1E-01 | 6.5E-04 | 6.6E-04 | 6.7E-04 | 8.7E-02 | -4.1E-01 |
| 20 | 20 | 2 | 5.0E-01 | 5.1E-01 | 5.0E-01 | 9.8E-01 | -4.0E-06 | 5.0E-04 | 4.9E-04 | 5.0E-04 | 9.8E-01 | -4.0E-06 |
| 20 | 15 | 2 | 5.0E-01 | 5.1E-01 | 5.0E-01 | 7.4E-01 | -2.4E-01 | 4.9E-04 | 4.9E-04 | 5.0E-04 | 7.4E-01 | -2.4E-01 |
| 20 | 10 | 2 | 5.0E-01 | 5.1E-01 | 5.0E-01 | 5.0E-01 | -4.8E-01 | 5.0E-04 | 4.9E-04 | 5.0E-04 | 5.0E-01 | -4.9E-01 |
| 20 | 5 | 2 | 5.0E-01 | 5.1E-01 | 5.0E-01 | 2.5E-01 | -7.2E-01 | 5.0E-04 | 4.9E-04 | 5.0E-04 | 2.6E-01 | -7.3E-01 |
| 20 | 20 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 6.6E-01 | -6.4E-01 | 6.5E-04 | 6.6E-04 | 6.7E-04 | 3.3E-01 | -3.2E-01 |
| 20 | 15 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 5.0E-01 | -8.0E-01 | 6.8E-04 | 6.6E-04 | 6.7E-04 | 2.5E-01 | -4.1E-01 |
| 20 | 10 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 3.4E-01 | -9.6E-01 | 6.7E-04 | 6.6E-04 | 6.7E-04 | 1.7E-01 | -4.9E-01 |
| 20 | 5 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 1.8E-01 | -1.1E+00 | 6.7E-04 | 6.6E-04 | 6.7E-04 | 8.9E-02 | -5.7E-01 |
| 25 | 25 | 2 | 5.0E-01 | 5.1E-01 | 5.0E-01 | 1.2E+00 | -9.5E-06 | 5.0E-04 | 4.9E-04 | 5.0E-04 | 1.2E+00 | -9.5E-06 |
| 25 | 20 | 2 | 5.0E-01 | 5.1E-01 | 5.0E-01 | 9.8E-01 | -2.4E-01 | 4.9E-04 | 4.9E-04 | 5.0E-04 | 9.8E-01 | -2.4E-01 |
| 25 | 15 | 2 | 5.0E-01 | 5.1E-01 | 5.0E-01 | 7.4E-01 | -4.8E-01 | 5.0E-04 | 4.9E-04 | 5.0E-04 | 7.5E-01 | -4.8E-01 |
| 25 | 10 | 2 | 5.0E-01 | 5.1E-01 | 5.0E-01 | 5.0E-01 | -7.1E-01 | 4.9E-04 | 4.9E-04 | 5.0E-04 | 5.0E-01 | -7.2E-01 |
| 25 | 5 | 2 | 5.0E-01 | 5.1E-01 | 5.0E-01 | 2.6E-01 | -9.5E-01 | 5.0E-04 | 4.9E-04 | 5.0E-04 | 2.6E-01 | -9.7E-01 |
| 25 | 25 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 8.3E-01 | -7.9E-01 | 6.6E-04 | 6.6E-04 | 6.7E-04 | 4.2E-01 | -4.0E-01 |
| 25 | 20 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 6.7E-01 | -9.5E-01 | 6.7E-04 | 6.6E-04 | 6.7E-04 | 3.4E-01 | -4.8E-01 |
| 25 | 15 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 5.0E-01 | -1.1E+00 | 6.7E-04 | 6.6E-04 | 6.7E-04 | 2.6E-01 | -5.7E-01 |
| 25 | 10 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 3.4E-01 | -1.3E+00 | 6.7E-04 | 6.6E-04 | 6.7E-04 | 1.7E-01 | -6.5E-01 |
| 25 | 5 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 1.8E-01 | -1.4E+00 | 6.7E-04 | 6.6E-04 | 6.7E-04 | 9.3E-02 | -7.3E-01 |

**Notes:** SKIP = The SKIP-CSP-1 plan, 1 = The CSP-1 plan, 2 = The CSP-2 plan,
SKIP-1 = Comparison of SKIP-CSP-1 plan with CSP-1 plan,
SKIP-2 = Comparison of the SKIP-CSP-1 with the CSP-2 plan.

From Tables 3, 4 and 5 it can be seen that for a very low level of defective value ($p$ = 0.001, 0.003) the $AFI$ values and $AOQ$ values for SKIP-CSP-1 are close to the values from the CSP-1 and CSP-2 plans for all values of $i, k, r$. At a high level of $p$ ($p$ = 0.05), the $AFI$ values for SKIP-CSP-1 are less than CSP-1 and more than CSP-2. For the $AOQ$ values, the SKIP-CSP-1 values are greater than CSP-1 values and less than CSP-2 values. However, for the higher level of $p$, the differences are relatively small for low values of $k$ ($k$ = 3, 5).

A comparison of the $AFI$ values for the three plans are shown in Figures 1, 2, 3, 4 and 5 for 3 different values of the defective probability $p$ (0.001, 0.003, 0.005, 0.01, 0.05) for a range of $i, k, r$ values. Figures 1, 2 and 3 show that the $AFI$ values for the 3 plans are approximately equal for a small probability $p$ = 0.001, 0.003, 0.005 of defectives (the actual errors for $p$ = 0.001, 0.003 are shown in Table 3 and Table 4). Figures 4 and 5 show that the differences between the $AFI$ values from the three plans become large as the value of $p$ is increased (the actual differences between the $AFI$ values at $p$ = 0.05 are given in Table 5).

**Table 4.** The $AFI$ values and $AOQ$ values for the SKIP-CSP-1, CSP-1 and CSP-2 plans for $p$ = 0.003

| $i$ | $k$ | $r$ | $AFI$ | | | $\%DIFF\_AFI$ | | $AOQ$ | | | $\%DIFF\_AOQ$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | SKIP | 1 | 2 | SKIP-1 | SKIP-2 | SKIP | 1 | 2 | SKIP-1 | SKIP-2 |
| 5 | 3 | 2 | 5.0E-01 | 5.0E-01 | 5.0E-01 | 4.5E-01 | -2.9E-01 | 1.5E-03 | 1.5E-03 | 1.5E-03 | 4.5E-01 | -2.9E-01 |
| 5 | 3 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 3.0E-01 | -6.8E-01 | 2.0E-03 | 2.0E-03 | 2.0E-03 | 1.5E-01 | -3.4E-01 |
| 10 | 10 | 2 | 5.0E-01 | 5.1E-01 | 5.0E-01 | 1.5E+00 | -2.1E-05 | 1.5E-03 | 1.5E-03 | 1.5E-03 | 1.5E+00 | -2.1E-05 |
| 10 | 5 | 2 | 5.0E-01 | 5.1E-01 | 5.0E-01 | 7.4E-01 | -7.1E-01 | 1.5E-03 | 1.5E-03 | 1.5E-03 | 7.5E-01 | -7.2E-01 |
| 10 | 10 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 9.9E-01 | -9.4E-01 | 2.0E-03 | 2.0E-03 | 2.0E-03 | 5.0E-01 | -4.8E-01 |
| 10 | 5 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 5.1E-01 | -1.4E+00 | 2.0E-03 | 2.0E-03 | 2.0E-03 | 2.6E-01 | -7.2E-01 |
| 15 | 15 | 2 | 5.0E-01 | 5.1E-01 | 5.0E-01 | 2.2E+00 | -9.8E-05 | 1.5E-03 | 1.5E-03 | 1.5E-03 | 2.2E+00 | -9.8E-05 |
| 15 | 10 | 2 | 5.0E-01 | 5.1E-01 | 5.0E-01 | 1.5E+00 | -6.9E-01 | 1.5E-03 | 1.5E-03 | 1.5E-03 | 1.5E+00 | -7.0E-01 |
| 15 | 5 | 2 | 5.1E-01 | 5.1E-01 | 5.0E-01 | 7.5E-01 | -1.4E+00 | 1.5E-03 | 1.5E-03 | 1.5E-03 | 7.7E-01 | -1.4E+00 |
| 15 | 15 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 1.5E+00 | -1.4E+00 | 2.0E-03 | 2.0E-03 | 2.0E-03 | 7.6E-01 | -7.0E-01 |
| 15 | 10 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 1.0E+00 | -1.8E+00 | 2.0E-03 | 2.0E-03 | 2.0E-03 | 5.2E-01 | -9.4E-01 |
| 15 | 5 | 3 | 3.4E-01 | 3.4E-01 | 3.3E-01 | 5.4E-01 | -2.3E+00 | 2.0E-03 | 2.0E-03 | 2.0E-03 | 2.8E-01 | -1.2E+00 |
| 20 | 20 | 2 | 5.0E-01 | 5.2E-01 | 5.0E-01 | 2.8E+00 | -2.9E-04 | 1.5E-03 | 1.5E-03 | 1.5E-03 | 2.8E+00 | -2.9E-04 |
| 20 | 15 | 2 | 5.0E-01 | 5.2E-01 | 5.0E-01 | 2.1E+00 | -6.7E-01 | 1.5E-03 | 1.5E-03 | 1.5E-03 | 2.2E+00 | -6.8E-01 |
| 20 | 10 | 2 | 5.1E-01 | 5.2E-01 | 5.0E-01 | 1.5E+00 | -1.3E+00 | 1.5E-03 | 1.5E-03 | 1.5E-03 | 1.5E+00 | -1.4E+00 |
| 20 | 5 | 2 | 5.1E-01 | 5.2E-01 | 5.0E-01 | 7.7E-01 | -2.0E+00 | 1.5E-03 | 1.5E-03 | 1.5E-03 | 8.0E-01 | -2.1E+00 |
| 20 | 20 | 3 | 3.4E-01 | 3.5E-01 | 3.3E-01 | 2.0E+00 | -1.8E+00 | 2.0E-03 | 2.0E-03 | 2.0E-03 | 1.0E+00 | -9.2E-01 |
| 20 | 15 | 3 | 3.4E-01 | 3.5E-01 | 3.3E-01 | 1.5E+00 | -2.2E+00 | 2.0E-03 | 2.0E-03 | 2.0E-03 | 7.8E-01 | -1.2E+00 |
| 20 | 10 | 3 | 3.4E-01 | 3.5E-01 | 3.3E-01 | 1.0E+00 | -2.7E+00 | 2.0E-03 | 2.0E-03 | 2.0E-03 | 5.4E-01 | -1.4E+00 |
| 20 | 5 | 3 | 3.4E-01 | 3.5E-01 | 3.3E-01 | 5.7E-01 | -3.1E+00 | 2.0E-03 | 2.0E-03 | 2.0E-03 | 3.0E-01 | -1.6E+00 |
| 25 | 25 | 2 | 5.0E-01 | 5.2E-01 | 5.0E-01 | 3.5E+00 | -6.8E-04 | 1.5E-03 | 1.4E-03 | 1.5E-03 | 3.5E+00 | -6.8E-04 |
| 25 | 20 | 2 | 5.0E-01 | 5.2E-01 | 5.0E-01 | 2.8E+00 | -6.5E-01 | 1.5E-03 | 1.4E-03 | 1.5E-03 | 2.9E+00 | -6.6E-01 |
| 25 | 15 | 2 | 5.1E-01 | 5.2E-01 | 5.0E-01 | 2.1E+00 | -1.3E+00 | 1.5E-03 | 1.4E-03 | 1.5E-03 | 2.2E+00 | -1.3E+00 |
| 25 | 10 | 2 | 5.1E-01 | 5.2E-01 | 5.0E-01 | 1.5E+00 | -1.9E+00 | 1.5E-03 | 1.4E-03 | 1.5E-03 | 1.5E+00 | -2.0E+00 |
| 25 | 5 | 2 | 5.1E-01 | 5.2E-01 | 5.0E-01 | 8.0E-01 | -2.6E+00 | 1.5E-03 | 1.4E-03 | 1.5E-03 | 8.4E-01 | -2.8E+00 |
| 25 | 25 | 3 | 3.4E-01 | 3.5E-01 | 3.3E-01 | 2.4E+00 | -2.2E+00 | 2.0E-03 | 1.9E-03 | 2.0E-03 | 1.3E+00 | -1.1E+00 |
| 25 | 20 | 3 | 3.4E-01 | 3.5E-01 | 3.3E-01 | 2.0E+00 | -2.6E+00 | 2.0E-03 | 1.9E-03 | 2.0E-03 | 1.0E+00 | -1.4E+00 |
| 25 | 15 | 3 | 3.4E-01 | 3.5E-01 | 3.3E-01 | 1.5E+00 | -3.0E+00 | 2.0E-03 | 1.9E-03 | 2.0E-03 | 8.0E-01 | -1.6E+00 |
| 25 | 10 | 3 | 3.5E-01 | 3.5E-01 | 3.3E-01 | 1.1E+00 | -3.5E+00 | 2.0E-03 | 1.9E-03 | 2.0E-03 | 5.7E-01 | -1.8E+00 |
| 25 | 5 | 3 | 3.5E-01 | 3.5E-01 | 3.3E-01 | 6.2E-01 | -3.9E+00 | 2.0E-03 | 1.9E-03 | 2.0E-03 | 3.3E-01 | -2.1E+00 |

**Table 5.** The *AFI* values and *AOQ* values for the SKIP-CSP-1, CSP-1 and CSP-2 plans for $p$ = 0.05

| *i* | *k* | *r* | *AFI* | | | *%DIFF_AFI* | | *AOQ* | | | *%DIFF_AOQ* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | SKIP | 1 | 2 | SKIP-1 | SKIP-2 | SKIP | 1 | 2 | SKIP-1 | SKIP-2 |
| 5 | 3 | 2 | 5.3E-01 | 5.6E-01 | 5.1E-01 | 6.3E+00 | -3.2E+00 | 2.3E-02 | 2.2E-02 | 2.4E-02 | 7.1E+00 | -3.6E+00 |
| 5 | 3 | 3 | 3.7E-01 | 3.9E-01 | 3.5E-01 | 5.2E+00 | -7.5E+00 | 3.1E-02 | 3.0E-02 | 3.3E-02 | 3.1E+00 | -4.5E+00 |
| 10 | 10 | 2 | 5.4E-01 | 6.3E-01 | 5.4E-01 | 1.4E+01 | -5.7E-01 | 2.3E-02 | 1.9E-02 | 2.3E-02 | 1.7E+01 | -6.9E-01 |
| 10 | 5 | 2 | 5.7E-01 | 6.3E-01 | 5.4E-01 | 8.4E+00 | -5.8E+00 | 2.2E-02 | 1.9E-02 | 2.3E-02 | 1.1E+01 | -7.9E+00 |
| 10 | 10 | 3 | 4.0E-01 | 4.6E-01 | 3.7E-01 | 1.2E+01 | -7.9E+00 | 2.9E-02 | 2.7E-02 | 3.1E-02 | 8.3E+00 | -5.4E+00 |
| 10 | 5 | 3 | 4.2E-01 | 4.6E-01 | 3.7E-01 | 7.9E+00 | -1.1E+01 | 2.8E-02 | 2.7E-02 | 3.1E-02 | 5.8E+00 | -8.4E+00 |
| 15 | 15 | 2 | 5.8E-01 | 6.8E-01 | 5.8E-01 | 1.5E+01 | -1.6E+00 | 2.0E-02 | 1.6E-02 | 2.1E-02 | 2.2E+01 | -2.4E+00 |
| 15 | 10 | 2 | 6.2E-01 | 6.8E-01 | 5.8E-01 | 1.1E+01 | -5.1E+00 | 2.0E-02 | 1.6E-02 | 2.1E-02 | 1.8E+01 | -8.1E+00 |
| 15 | 5 | 2 | 6.4E-01 | 6.8E-01 | 5.8E-01 | 7.1E+00 | -8.5E+00 | 1.8E-02 | 1.6E-02 | 2.1E-02 | 1.2E+01 | -1.5E+01 |
| 15 | 15 | 3 | 4.5E-01 | 5.2E-01 | 4.1E-01 | 1.4E+01 | -9.6E+00 | 2.7E-02 | 2.4E-02 | 2.9E-02 | 1.2E+01 | -8.0E+00 |
| 15 | 10 | 3 | 4.6E-01 | 5.2E-01 | 4.1E-01 | 1.1E+01 | -1.2E+01 | 2.6E-02 | 2.4E-02 | 2.9E-02 | 9.5E+00 | -1.1E+01 |
| 15 | 5 | 3 | 4.8E-01 | 5.2E-01 | 4.1E-01 | 7.7E+00 | -1.4E+01 | 2.6E-02 | 2.4E-02 | 2.9E-02 | 7.1E+00 | -1.3E+01 |
| 20 | 20 | 2 | 6.5E-01 | 7.4E-01 | 6.3E-01 | 1.4E+01 | -2.9E+00 | 1.8E-02 | 1.3E-02 | 1.9E-02 | 2.5E+01 | -5.4E+00 |
| 20 | 15 | 2 | 6.5E-01 | 7.4E-01 | 6.3E-01 | 1.1E+01 | -5.2E+00 | 1.6E-02 | 1.3E-02 | 1.9E-02 | 2.1E+01 | -1.0E+01 |
| 20 | 10 | 2 | 6.8E-01 | 7.4E-01 | 6.3E-01 | 8.2E+00 | -7.4E+00 | 1.7E-02 | 1.3E-02 | 1.9E-02 | 1.8E+01 | -1.6E+01 |
| 20 | 5 | 2 | 6.9E-01 | 7.4E-01 | 6.3E-01 | 5.6E+00 | -9.7E+00 | 1.5E-02 | 1.3E-02 | 1.9E-02 | 1.3E+01 | -2.2E+01 |
| 20 | 20 | 3 | 5.1E-01 | 5.8E-01 | 4.6E-01 | 1.3E+01 | -1.1E+01 | 2.3E-02 | 2.1E-02 | 2.7E-02 | 1.4E+01 | -1.2E+01 |
| 20 | 15 | 3 | 5.2E-01 | 5.8E-01 | 4.6E-01 | 1.1E+01 | -1.2E+01 | 2.4E-02 | 2.1E-02 | 2.7E-02 | 1.2E+01 | -1.4E+01 |
| 20 | 10 | 3 | 5.4E-01 | 5.8E-01 | 4.6E-01 | 8.9E+00 | -1.4E+01 | 2.3E-02 | 2.1E-02 | 2.7E-02 | 1.0E+01 | -1.6E+01 |
| 20 | 5 | 3 | 5.3E-01 | 5.8E-01 | 4.6E-01 | 6.8E+00 | -1.6E+01 | 2.4E-02 | 2.1E-02 | 2.7E-02 | 8.2E+00 | -1.9E+01 |
| 25 | 25 | 2 | 7.0E-01 | 7.8E-01 | 6.8E-01 | 1.1E+01 | -4.1E+00 | 1.5E-02 | 1.1E-02 | 1.6E-02 | 2.6E+01 | -9.9E+00 |
| 25 | 20 | 2 | 7.1E-01 | 7.8E-01 | 6.8E-01 | 9.3E+00 | -5.6E+00 | 1.5E-02 | 1.1E-02 | 1.6E-02 | 2.3E+01 | -1.4E+01 |
| 25 | 15 | 2 | 7.3E-01 | 7.8E-01 | 6.8E-01 | 7.6E+00 | -7.0E+00 | 1.4E-02 | 1.1E-02 | 1.6E-02 | 2.0E+01 | -1.9E+01 |
| 25 | 10 | 2 | 7.4E-01 | 7.8E-01 | 6.8E-01 | 5.9E+00 | -8.5E+00 | 1.3E-02 | 1.1E-02 | 1.6E-02 | 1.7E+01 | -2.4E+01 |
| 25 | 5 | 2 | 7.5E-01 | 7.8E-01 | 6.8E-01 | 4.2E+00 | -9.9E+00 | 1.3E-02 | 1.1E-02 | 1.6E-02 | 1.3E+01 | -3.0E+01 |
| 25 | 25 | 3 | 5.7E-01 | 6.4E-01 | 5.1E-01 | 1.1E+01 | -1.2E+01 | 2.1E-02 | 1.8E-02 | 2.4E-02 | 1.5E+01 | -1.6E+01 |
| 25 | 20 | 3 | 5.9E-01 | 6.4E-01 | 5.1E-01 | 9.7E+00 | -1.3E+01 | 2.0E-02 | 1.8E-02 | 2.4E-02 | 1.4E+01 | -1.8E+01 |
| 25 | 15 | 3 | 6.0E-01 | 6.4E-01 | 5.1E-01 | 8.3E+00 | -1.4E+01 | 2.0E-02 | 1.8E-02 | 2.4E-02 | 1.2E+01 | -2.0E+01 |
| 25 | 10 | 3 | 6.0E-01 | 6.4E-01 | 5.1E-01 | 6.9E+00 | -1.5E+01 | 2.0E-02 | 1.8E-02 | 2.4E-02 | 1.0E+01 | -2.3E+01 |
| 25 | 5 | 3 | 6.3E-01 | 6.4E-01 | 5.1E-01 | 5.5E+00 | -1.6E+01 | 1.7E-02 | 1.8E-02 | 2.4E-02 | 8.6E+00 | -2.5E+01 |

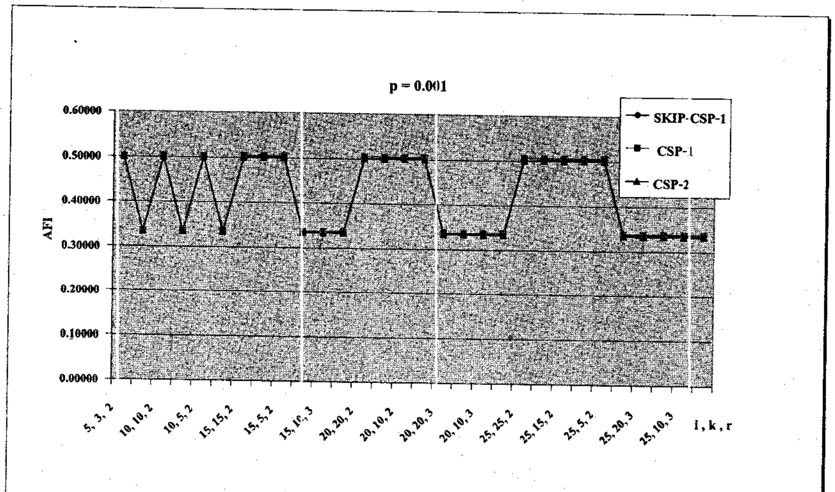


**Figure 1.** A comparison of *AFI* values for the SKIP-CSP-1, CSP-1 and CSP-2 plans for $p$ = 0.001 for a range of $i, k, r$ values.

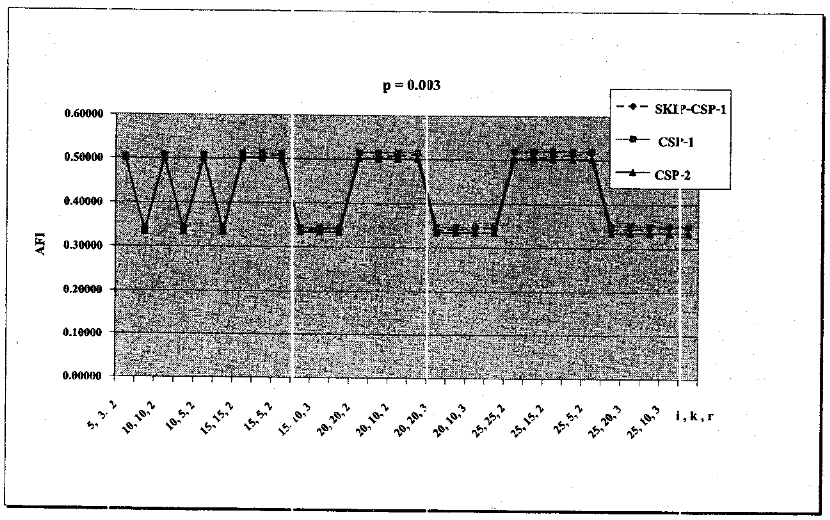


**Figure 2.** A comparison of $AFI$ values for the SKIP-CSP-1, CSP-1 and CSP-2 plans for $p = 0.003$ for a range of $i, k, r$ values.

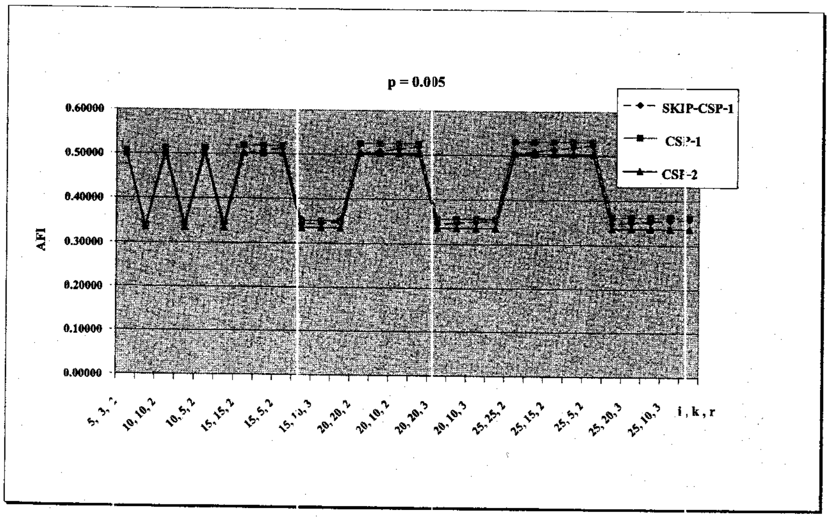


**Figure 3.** A comparison of $AFI$ values for the SKIP-CSP-1, CSP-1 and CSP-2 plans for $p = 0.005$ for a range of $i, k, r$ values.

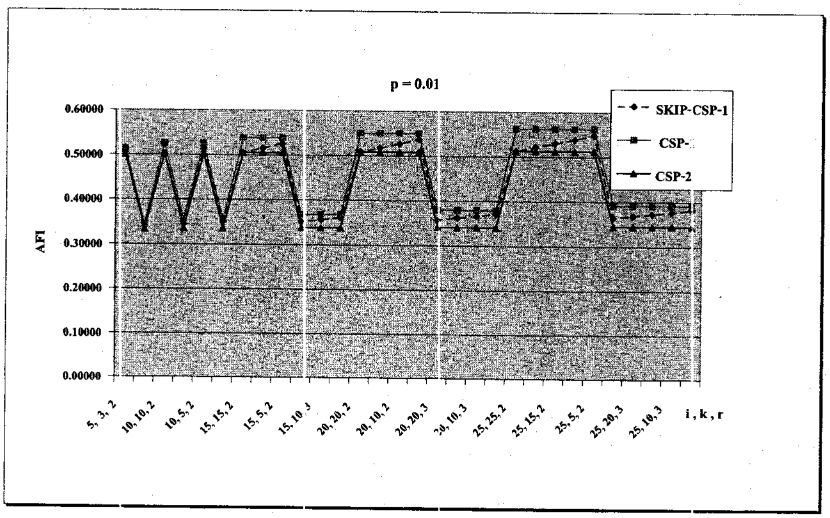


**Figure 4.** A comparison of $AFI$ values for the SKIP-CSP-1, CSP-1 and CSP-2 plans for $p = 0.01$ for a range of $i, k, r$ values.

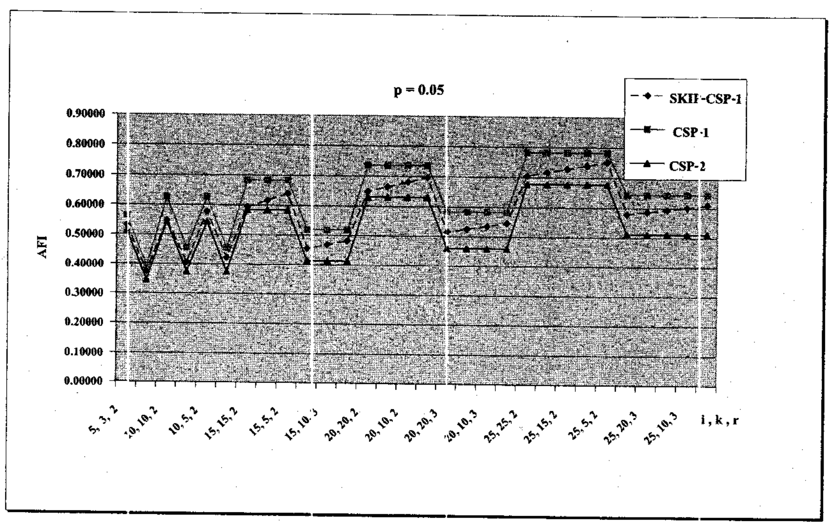


**Figure 5.** A comparison of $AFI$ values for the SKIP-CSP-1, CSP-1 and CSP-2 plans for $p = 0.05$ for a range of $i, k, r$ values.

## 4. Discussion and Conclusions

A new SKIP-CSP-1 plan has been developed which was originally designed for inspection of high-quality production lines. The new plan differs from the original CSP-1 plan in that a specified number of units are not inspected (skipped over) if no defective units are found during an initial 100% inspection. Formulas for the average fraction inspected ( $AFI$ ), average outgoing quality ( $AOQ$ ) and average outgoing quality limit (*AOQL*) have been developed.

The validity of the formulas for the new method has been tested by extensive simulations for a range of values of $p$ (probability of defective units), $i$ (number of consecutive non-defective units required in 100% inspections), $k$ (number of units not inspected after no defective units found in 100% inspection) and $r$ (number of units produced between inspections in a fractional inspection). The simulated and formula values were found to agree within 2% in all simulations.

Extensive simulations have been carried out to compare the $AFI$ and $AOQ$ values obtained from the SKIP-CSP-1 plan with $AFI$ and $AOQ$ values from CSP-1 and CSP-2 plans. The three plans were found to give approximately equal values for a low probability of defective units ( $p$ = 0.001, 0.003) and for higher probabilities when the values of $k$ and $r$ were small. However, there were appreciable differences between the values in other cases, i.e., for larger $p$, $k$ and $r$ values, with the $AFI$ values increasing in the order CSP-2, SKIP-CSP-1, CSP-1.

The results show that for high-quality lines ( $p$ = 0.001, 0.003, 0.005) an operator may choose to use any of the 3 plans and know that the $AFI$ values and $AOQ$ values will be the same. One advantage of the SKIP-CSP-1 plan is that the operator knows that if a 100% inspection is successful, then they do not have to inspect the line for another fixed number of units. In contrast, the other 2 plans may require switching between a fractional inspection and a 100% inspection at any time. For higher probabilities of defectives, the SKIP-CSP-1 and CSP-1 plans give approximately the same $AFI$ and $AOQ$ values for small skip values ( $k$ □ $i$ ), whereas the SKIP-CSP-1 and CSP-2 plans give approximately the same $AFI$ and $AOQ$ values for large skip values ( $k \cong i$ ). Since we believe that the SKIP-CSP-1 plan is easier to implement and more predictable in inspection times, an operator should always consider using the new plan in preference to a CSP-2 plan. An operator should consider using the SKIP-CSP-1 plan in preference to a CSP-1 plan for high quality lines and even on lines with large $p$ (0.008 to 0.01) provided the skip values $k$ are not close to $i$ values. However, the results also show that the highest average quality output will always be obtained with the CSP-1 plan.

## 5. Acknowledgements

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